Section 4.3
Connecting f' and f" with the Graph of f

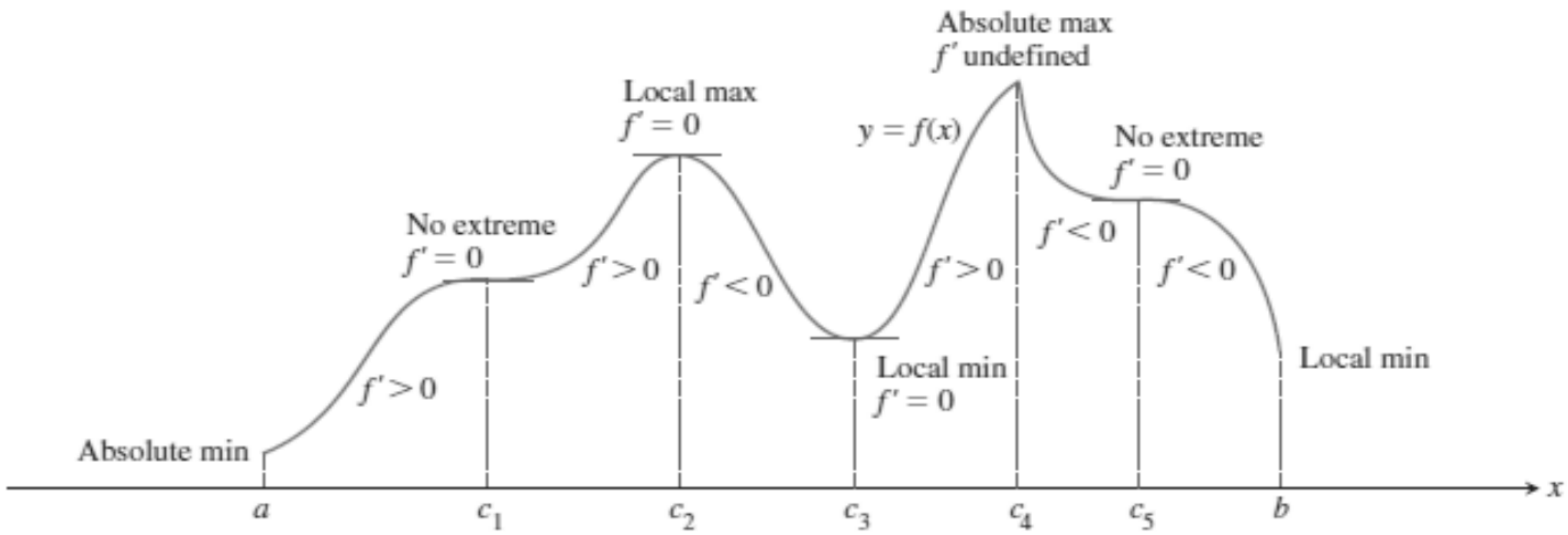


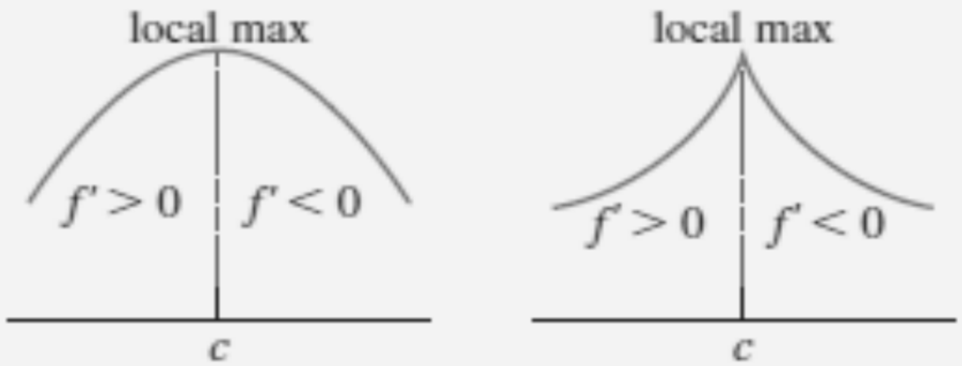
Figure 4.18 A function's first derivative tells how the graph rises and falls.

First Derivative Test for Local Extrema

The following test applies for a continuous function

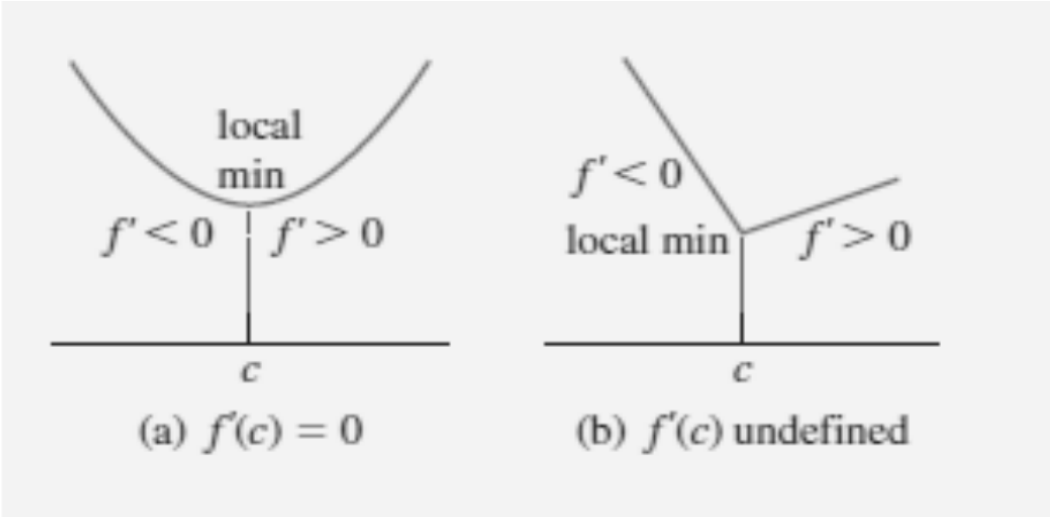
At a critical point c:

1.) If f' changes sign from positive to negative at c (f'>0 for x < c and f'<0 for x>c), then f has a local maximum value at c.

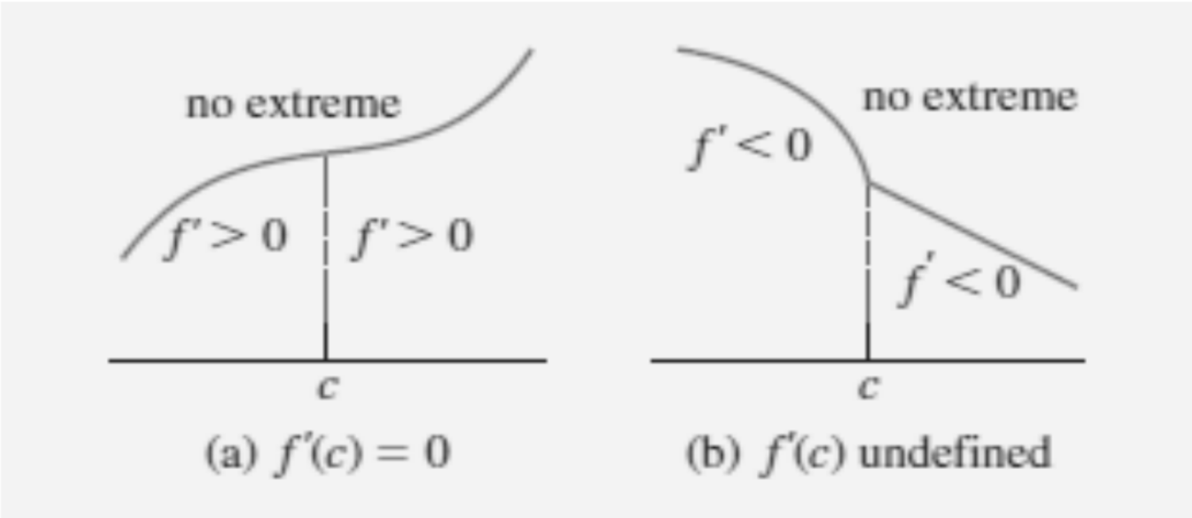


(a) $f'(c) = 0$ (b) $f'(c)$ undefined

2.) If f' changes sign from negative to positive at c (f' < 0 for x < c and f' > c for x > c, then f has a local minimum value at c.

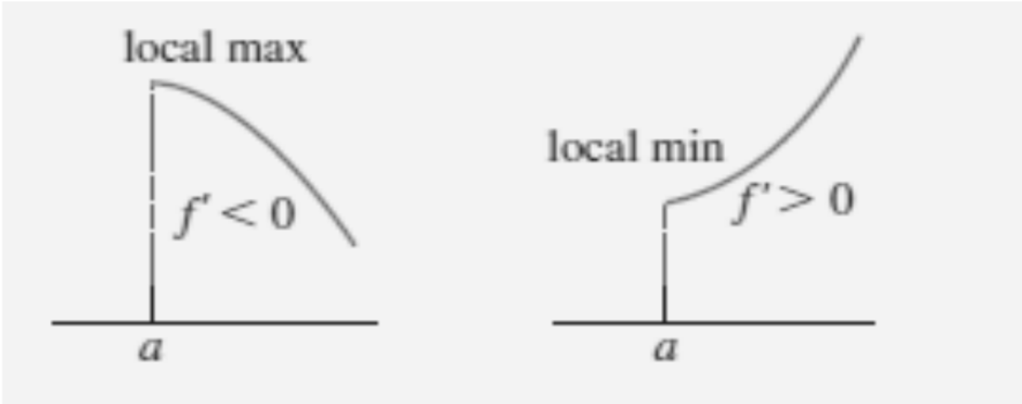


3.) If f' does not change sign at c (f' has the same sign on both sides of c), then f has no local extreme values at c.



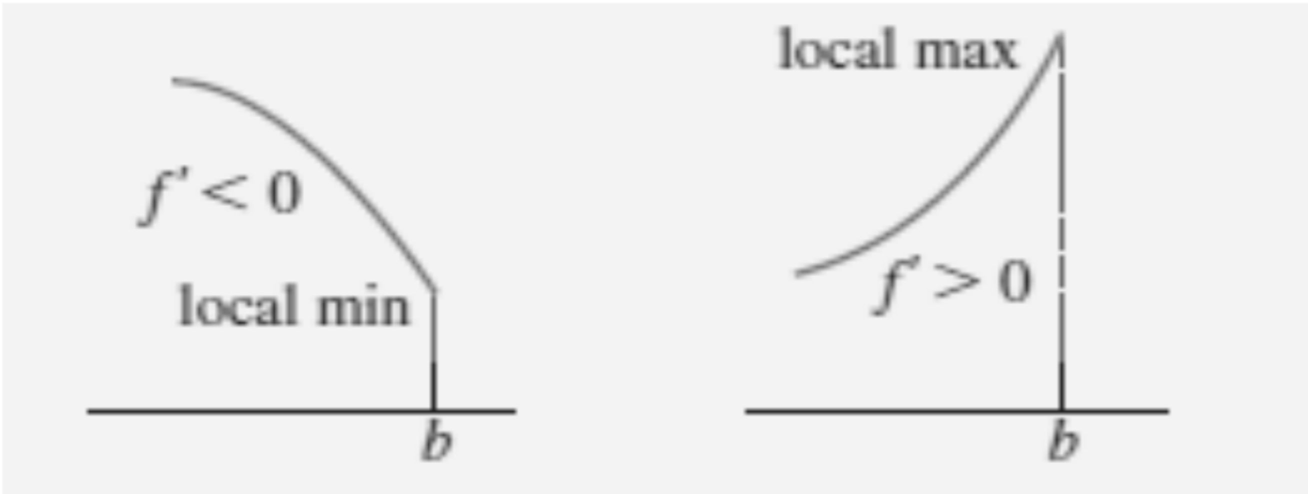
At a left endpoint a:

If f' < 0(f'>0) for x > a, then f has a local maximum (minimum) value at a



At a right endpoint b:

If f'<0 (f' > 0) for x < b, then f has a local minimum (maximum) value at b.



For each of the following functions, use the First Derivative Test to find the local extreme values. Identify any absolute extrema.

a.) $f(x) = x^3 - 12x - 5$

b.) $g(x) = (x^2 - 3)e^x$

Use the first derivative test to determine the local extreme values of the function, and identify any absolute extrema.

$$y = 2x^4 - 4x^2 + 1$$

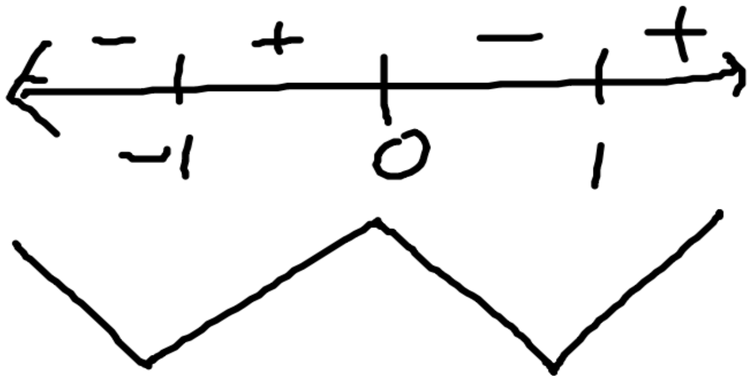
$$y' = 8x^3 - 8x$$

$$8x^3 - 8x = 0$$

$$8x(x^2 - 1) = 0$$

$$8x = 0 \qquad x^2 - 1 = 0$$

$$x = 0 \qquad x = \pm 1$$



Abs. min @ (−1, −1)

Abs. min @ (1, −1)

local max @ (0, 1)

Concavity

The graph of a differentiable function y = f(x) is

a.) concave up on an open interval *I* if y' is increasing on *I*.

b.) concave down on an open interval *I* if y' is decreasing on *I*.

Concavity Test

The graph of a twice - differentiable function $y = f(x)$ is

Second Derivative

a.) concave up on any interval where $y'' > 0$.

b.) concave down on any interval where $y'' < 0$.

Use the concavity test to determine the concavity of the given functions on the given intervals

a.) $y = x^2$ on $(3,10)$

$y' = 2x$

$y'' = 2$

Always C. Up.

b.) $y = 3 + \sin x$ on $(0, 2\pi)$

$y' = \cos x$

$y'' = -\sin x$

$-\sin x = 0$

Points of Inflection

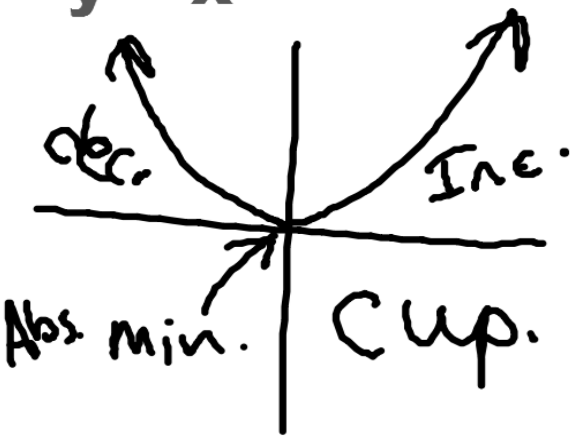
The curve $y = 3 + \sin x$ changes concavity at the point $(\pi, 3)$. This is a point of **inflection** of the curve.

A point where the graph of a function has a tangent line and where the concavity changes.

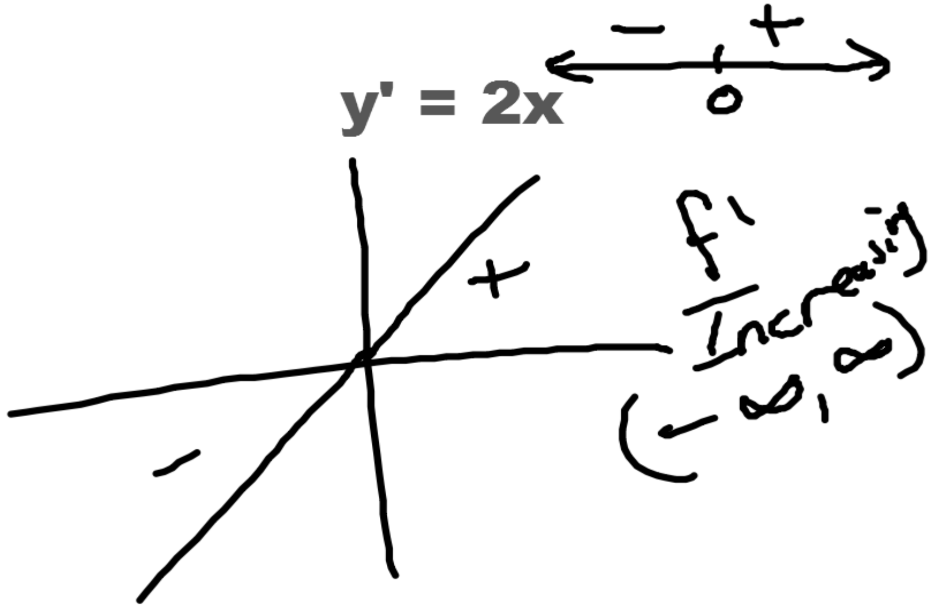
A point on a curve where y'' is positive on one side and negative on the other is a point of inflection.

At such a point, y'' is either zero or undefined!

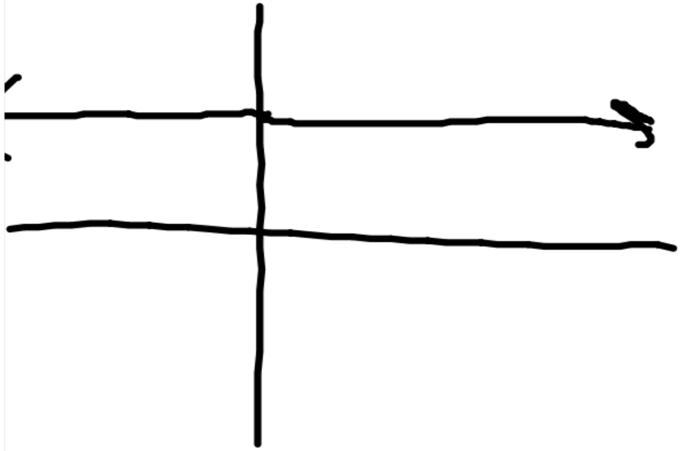
$y = x^2$



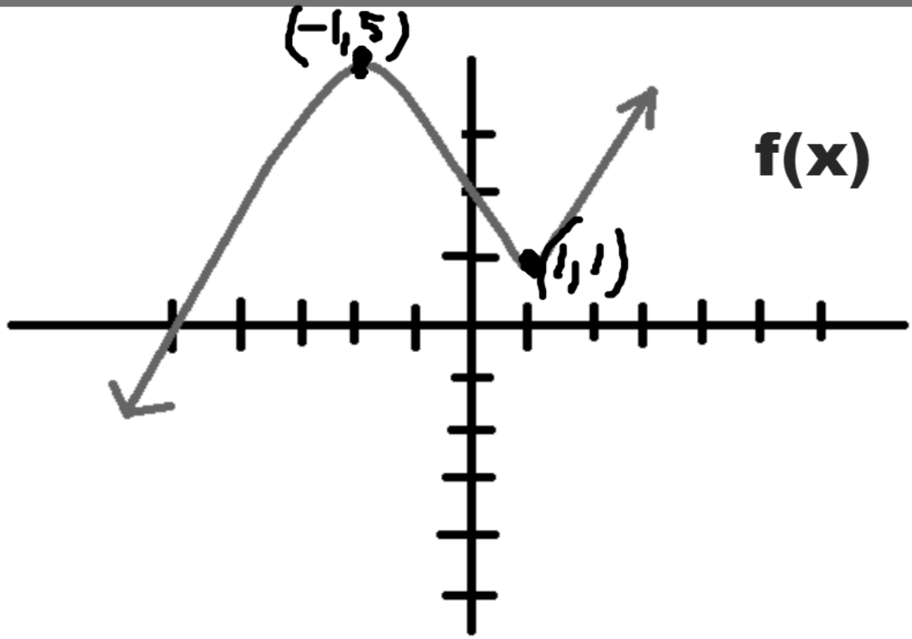
$y' = 2x$



$y'' = 2$



f	f'	f''
inc	+	
dec	−	
max/min	ZERO/undef.	
c. up		+
c. down		−



f(x)

$f'(x) > 0$

$(-\infty, -1) \cup (1, \infty)$

$f'(x) < 0$

$(-1, 1)$

$f''(x) > 0$

$f''(x) < 0$

Find all points of inflection of the graph of

$$y = e^{-x^2}$$

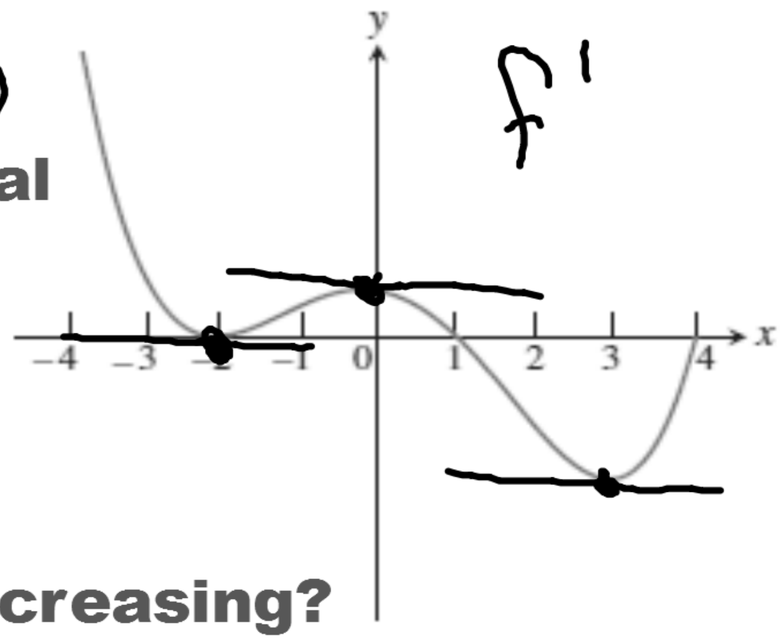
$$y' = e^{-x^2} \cdot (-2x)$$

$$y' = -2x e^{-x^2}$$

$$y'' = (-2)(e^{-x^2}) + (-2x)(e^{-x^2})(-2x)$$

$$y'' = -2e^{-x^2} + 4x^2 e^{-x^2}$$

The graph of the derivative of a function f on the interval [-4,4] is shown. Answer the following questions about f.



a.) On what intervals is f increasing?

$f'(x) > 0$

b.) On what intervals is the graph of f concave up?

c.) At which x-coordinate does f have local extrema?

d.) What are the x-coordinates of all inflection points of the graph of f?

$f(x) = x^3 - 6x^2 + 5$

Extrema.

Inc, dec.

C.Up

C.Down.

POI

pg. 215

#1, 2, 3

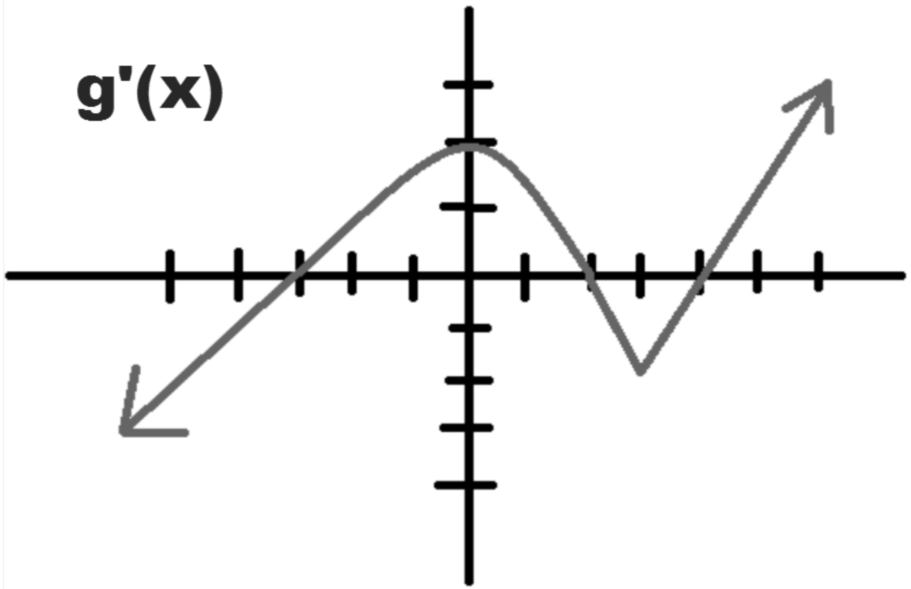
21, 23

In order to have a maximum or minimum or point of inflection, there must be a sign change at the critical point!

$$y = x^3$$

Page 215, numbers 1,2,3 (Find inc,dec,concave up, concave down, max/min, POI
MUST USE f' and f"

and numbers 21, 23


g'(x)

$f(x) = x^2 - x - 1$

Find inc, dec, max, min, poi, concave up, down, etc.

$f(x) = -2x^3 + 6x^2 - 3$

Find inc, dec, max, min, poi, concave up, down, etc.

$y = 2x^4 - 4x^2 + 1$

Find inc, dec, max, min, poi, concave up, down, etc.

Second Derivative test for Local Extrema

Instead of looking for sign changes in y' at critical points, we can sometimes use the following test to determine the presence of local extrema.

Second Derivative Test for Local Extrema

1.) If $f'(c) = 0$ and $f''(c) < 0$, then f has a local maximum at $x = c$.

2.) If $f'(c)$ and $f''(c) > 0$, then f has a local minimum at $x = c$.

3.) If $f''(c) = 0$, it is inconclusive

$$y = 2x^2 - x$$

f(x) has a horizontal tangent at $x = 4$.

$f''(x) = 3x^2 - 4x$. Does f(x) have a max or min at $x = 4$?

Find the local extreme values of $f(x) = x^3 - 12x - 5$.

Let $f'(x) = 4x^3 - 12x^2$

a.) Identify where the extrema of f occur

b.) Find the intervals on which f is increasing and the intervals on which f is decreasing.

c.) Find where the graph of f is concave up and where it is concave down.

$y = e^{-x}$ $-1 \leq x \leq 1$

$y = x^{2/5}$ $-3 \leq x < 1$

page 218, numbers 55 - 60

Find local / absolute extrema :

$y = \ln(x + 1) \qquad 0 \leq x \leq 3$

$$y = \begin{cases} 4 - 2x & x \leq 1 \\ x + 1 & x > 1 \end{cases}$$

$$y = \begin{cases} 3 - x & x < 0 \\ 3 + 2x - x^2 & x \geq 0 \end{cases}$$

Find all points of inflection of :

$y = xe^x$

$$y = \begin{cases} 3x^2 - 4x + 1 & x \geq 0 \\ 5x - 2x^2 + 1 & x < 0 \end{cases}$$